

Entanglement between charge qubits induced by a common dissipative environment

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(Dated: February 2, 2008)

We study entanglement generation between two charge qubits due to the strong coupling with a common bosonic environment (Ohmic bath). The coupling to the boson bath is a source of both quantum noise (leading to decoherence) and an indirect interaction between qubits. As a result, two effects compete as a function of the coupling strength with the bath: entanglement generation and charge localization induced by the bath. These two competing effects lead to a non-monotonic behavior of the concurrence as a function of the coupling strength with the bath. As an application, we present results for charge qubits based on double quantum dots.

PACS numbers: 03.65.Ud, 03.67.Mn, 73.21.La, 73.40.Gk, 73.63.Kv, 85.35.Be

I. INTRODUCTION

Solid state nanostructures have become promising candidates for quantum information processing¹, with basic operations like single-qubit manipulation and read-out having been demonstrated during the last few years. However, to go beyond single-qubit manipulations, and study effects such as entanglement generation and quantum gate operations, one needs some kind of interaction between the qubits. Although this interaction usually comes from a direct coupling between qubits (like the Coulomb interaction for charge qubits or exchange coupling for spin qubits), entanglement can be also generated by coupling two qubits (which do not interact with each other) to a common third system^{2,3,4,5,6,7,8,9,10,11}. In most of these studies, the indirect interaction comes from the coupling to one or a few external degrees of freedom. Examples include the coupling to electromagnetic modes in a cavity (see, for example, Ref. 2, where the authors study entanglement of atoms within a single-mode cavity field) or to a harmonic oscillator representing a mode in a thermal environment^{10,11}. Importantly, entanglement can be also induced when the environment is made by an infinitely large number of degrees of freedom, namely a bath, as demonstrated by Braun in Ref. 12. This is an important case because entanglement is generated exclusively by *incoherent means*. In this context, different works have studied the coupling of two non-interacting qubits to fermionic^{13,14,15,16,17} or bosonic^{11,12,18,19,20} baths.

Indirect qubit interactions have attracted attention recently because the information distribution among distant entangled particles is the base of quantum cryptography²¹, quantum teleportation^{22,23,24} quantum dense code^{24,25,26}, different processes proposed for testing Bell inequalities^{27,28,29,30} and even certain steps within quantum computation algorithms. The possibility of entangling two quantum systems which do not interact directly is therefore highly desirable, with various aspects of current interest like "entanglement

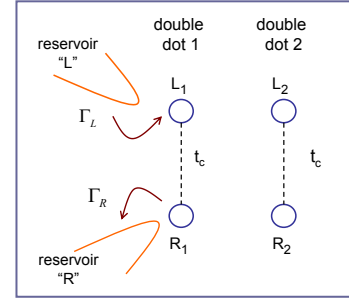


FIG. 1: Two independent double quantum dots coupled to a common phonon environment. Interdot hopping, t_c , is allowed only in each double dot. The first double quantum dot is open to electron reservoirs, with probabilities for tunneling in and out given by Γ_L and Γ_R , respectively.

swapping"^{27,31,32,33,34} and "entanglement transfer"^{35,36}.

In this paper we study entanglement generation between two charge qubits due to the strong coupling with a common bosonic environment (Ohmic bath). For concreteness, we focus on charge qubits based on double quantum dots (DQDs) but we point out, that our results can also be applied to Cooper Pair Boxes in a resistive environment. In a DQD, the electron charge degree of freedom is used to construct a qubit^{37,38,39,40,41,42}, with logical states $|0\rangle$ and $|1\rangle$ corresponding to the localization of one excess electron on each one of the quantum dots (QD). One of the advantages of these charge qubits is their controllability through external voltages handling, as demonstrated in recent experiments³⁹ where the charge has been coherently manipulated. We model the two DQD system as two independent two-level systems strongly coupled to the same Ohmic bath (two spin-boson models). In addition, we consider that one of the DQDs is coupled to electron reservoirs⁴³ (Fig. 1), in order to allow

electronic transport. The coupling to electronic reservoirs is treated using a Markovian approach^{42,43,44,45}, which is valid in the sequential tunneling limit and large bias voltages. The non-Markovian character of the strong coupling with the boson bath is, on the other hand, taken into account by using a polaron approach^{42,43,44,46,47}. As a result of this strong coupling, an indirect Ising-like interaction between qubits is induced by the bath.

By combining both Markovian and non-Markovian approximations, we derive a master equation for the reduced density matrix of the system, including boson correlation functions in Laplace space. The resulting density matrix is used to calculate the degree of entanglement (quantified by Wotters' concurrence⁴⁸) as well as the probability for each one of the Bell states as a function of the coupling strength by the bath.

Our results complement previous work by Vorrath and Brandes in Ref. 20 who studied a similar problem within a Markovian approach. We also mention some recent works^{11,18,19} in which related models are treated.

The paper is organized as follows: in section II the model describing the DQD coupled to both, electronic reservoirs and the bosonic environment is discussed. We also present in Section II the general solution scheme for the density matrix equations. The coupling with the leads is treated by using a standard Born-Markov approximation whereas the strong coupling with the bath is treated within a polaron approach. Section III shows the main results obtained, and finally we conclude in section IV.

II. MODEL

An array of two parallel DQDs in the strong Coulomb Blockade regime, and coupled to the same bosonic environment, is considered. Interdot tunneling is allowed only in each double dot, defining an array of two charge qubits (Fig. 1). The first DQD is weakly coupled to two electron reservoirs in such way that electronic transport through this double dot is possible (the excess charge in this DQD fluctuates between one and zero). The second DQD is closed and, therefore, has always one excess electron. Note that such configuration is close to the one realized in very recent experiments⁴⁹. The Hilbert space includes two-particles states $|1\rangle = |L_1 L_2\rangle$, $|2\rangle = |L_1 R_2\rangle$, $|3\rangle = |R_1 L_2\rangle$ and $|4\rangle = |R_1 R_2\rangle$ (where $L_i(R_i)$ represents the charge localized in the upper (lower) QD of the i -th DQD), as well as one particle states $|5\rangle = |0_1 L_2\rangle$ and $|6\rangle = |0_1 R_2\rangle$ (where 0 means no extra electron in the first DQD). The completeness of the system is therefore given by $\sum_{k=1}^6 |k\rangle\langle k|$.

The total Hamiltonian describing this system reads

$$H = H_{sys} + H_{res} + H_B + H_{SR} + H_{SB}. \quad (1)$$

The free part of the Hamiltonian, i.e without couplings, contains three terms. The first term corresponds to the

Hamiltonian of two independent DQDs, which in pseudo-spin language can be written as

$$H_{sys} = \sum_i^2 \frac{1}{2} \Delta \varepsilon_i \sigma_z^i + t_c \sigma_x^i, \quad (2)$$

where $\Delta \varepsilon_i$ is the energy difference between quantum dots of each pair being $\Delta \varepsilon_1 = \varepsilon_{L_1} - \varepsilon_{R_1}$ and $\Delta \varepsilon_2 = \varepsilon_{L_2} - \varepsilon_{R_2}$ (ε_{ij} is the on-site energy of the i -th QD of the pair j), σ_j^i is the j -th Pauli matrix acting on each DQD, t_c is the electron tunneling amplitude which is considered identical for both DQDs^{43,44}.

The Hamiltonian of the reservoirs, referred as "L" and "R", reads^{42,43,44}

$$H_{res} = \sum_k \{ \epsilon_k^L c_{k,L}^\dagger c_{k,L} + \epsilon_k^R c_{k,R}^\dagger c_{k,R} \}, \quad (3)$$

where $c_{k,\beta}^\dagger$ and $c_{k,\beta}$ are fermion creation and annihilation operators in lead β with corresponding energy ϵ_k^β . Finally, the third term corresponds to the boson bath, which is described as a set of harmonic oscillators with frequency ω_q :

$$H_B = \sum_q \hbar \omega_q a_q^\dagger a_q \quad (4)$$

$a_q^\dagger(a_q)$ is the annihilation (creation) boson operator^{43,50}.

As we have mentioned already, we take into account the coupling of the system to both electronics reservoirs and a boson bath. The first coupling is given by

$$H_{SR} = \sum_{k,i \in 1,2} \{ V_k^L (c_{k,L}^\dagger s_{L,i} + c.c.) + V_k^R (c_{k,R}^\dagger s_{R,i} + c.c.) \}, \quad (5)$$

with V_k^β being the coupling with the lead β . The Lindblad-type operators $s_{L,i}$ ($s_{R,i}$) describe tunneling into (out of) the first DQD taking into account the two possible configurations ($\sum_{i \in 1,2}$) in the second DQD, namely $s_{L,1} = |5\rangle\langle 1|$, $s_{L,2} = |6\rangle\langle 2|$, $s_{R,1} = |5\rangle\langle 3|$ and $s_{R,2} = |6\rangle\langle 4|$. Although we consider the coupling of only one of the DQDs to reservoirs, the generalization for both double dots is straightforward.

The electron-boson interaction is described with a spin-boson Hamiltonian, where the bath "force" operator $\xi^i = \sum_q \gamma_q^i (a_q^\dagger + a_q)$ couples linearly to each qubit's $\sigma_z^{i50,51}$ (here we consider that such interaction is identical for both DQDs,⁵² $\gamma_q^i = \gamma_q$):

$$H_{SB} = \sum_q \sum_i \frac{1}{2} \gamma_q \sigma_z^i (a_q^\dagger + a_q), \quad (6)$$

Note that this coupling which is longitudinal in the local basis of each qubit, contains both longitudinal and traversal components in the basis that diagonalizes the

qubit Hamiltonian⁵³. The bath effects can be encapsulated in the spectral density $J(\omega) = \sum_q \gamma_q^2 \delta(\omega - \omega_q)$. In the following, we use a generic ohmic bath: $J(\omega) = 2\alpha\omega e^{-\omega/\omega_c}$, where ω_c is a cutoff frequency and α is a dimensionless parameter which reflects the dissipation strength^{50,51,54}. As we shall see in the next subsection, the coupling of both qubits to the same quantum heat bath leads to decoherence *and* to an effective interaction between qubits.

A. Polaron Transformation

Due to the strong coupling of the qubits with the boson bath, a proper description of the system must take into account non-Markovian effects. Among the different approaches available to deal with this^{55,56,57,58}, we use a "polaron transformation"⁴⁷, which for an arbitrary operator O is given by:

$$\begin{aligned}\bar{O} &= e^S O e^{-S}, \\ S &= \sum_q \sum_i \frac{1}{2} \sigma_z^i \frac{\gamma_q}{\omega_q} (a_q^\dagger - a_q).\end{aligned}\quad (7)$$

This approach, which is well-known for treating problems in which bosonic modes couple to localized electronic states, has been successfully used for studying single^{59,60} and double quantum dots^{42,43,44,46} strongly coupled to a bath of phonons.

Applying the transformation in Eq. (7) to the relevant operators in our model, we obtain the transformed operators:

$$\begin{aligned}\bar{\sigma}_z^i &= \sigma_z^i \\ \bar{\sigma}_x^i &= \sigma_+^i X + \sigma_-^i X^\dagger \\ \bar{a}_q &= a_q - \frac{1}{2} \frac{\lambda_q}{\omega_q} \sum_i \sigma_z^i \\ \bar{s}_{L,i} &= s_{L,i} e^{-A/2} \\ \bar{s}_{R,i} &= s_{R,i} e^{A/2}\end{aligned}\quad (8)$$

where σ_+^i and σ_-^i are the ladder spin operators on each DQD. The operators X and X^\dagger are polaronic phases⁴⁷ given by

$$X = e^A, \quad (9)$$

with

$$A = \sum_q \gamma_q (a_q^\dagger - a_q). \quad (10)$$

By substituting these *transformed* operators into the equations (2) to (6), we obtain the effective Hamiltonian:

$$\bar{H} = \bar{H}_0 + \bar{H}_T + \bar{H}_{SR} \quad (11)$$

$$\bar{H}_0 = \sum_i \frac{1}{2} \Delta \varepsilon_i \sigma_z^i - \frac{1}{4} \kappa \sum_{i,j} \sigma_z^i \sigma_z^j + H_B + H_{res} \quad (12)$$

$$\bar{H}_T = \sum_i t_c (\sigma_+^i X + \sigma_-^i X^\dagger) \quad (13)$$

The effect of the canonical transformation is threefold:

(i) The electron-boson interaction H_{SB} has been transformed away.

(ii) The state of the bosonic system is strongly modified every time an electron tunnels between dots (boson "shake-up")⁶¹. As a result, the interdot tunneling amplitude (Eq. 13) becomes renormalized with environment-dependent phases through the operators $X = e^A$. These time-dependent exponential phases, which appear as a result of the non-perturbative treatment of the electron-boson interaction, lead to non-trivial effects. In particular, this implies that non-Markovian effects become relevant and need to be considered in the dynamics of the reduced density matrix. Note also that, in principle, this renormalization of tunneling has to be taken into account also in \bar{H}_{SR} through the operators $\bar{s}_{L,i} = s_{L,i} e^{-A/2}$ and $\bar{s}_{R,i} = s_{R,i} e^{A/2}$. However, this is no longer true in the limit of large bias voltages, where the coupling to the reservoirs becomes Markovian (see the next subsection).

(iii) The transformed Hamiltonian contains an *effective interaction* between qubits $H_{eff} = -\frac{\kappa}{4} \sum_{i,j} \sigma_z^i \sigma_z^j$ due to the coupling with the common bath; this interaction has an Ising form and depends on the parameter $\kappa = \sum_q \gamma_q^2 / \omega_q$ (which for Ohmic dissipation used here reads $\kappa = 2\alpha\omega_c$) favoring states with the same charge distribution in both DQDs.

B. Master equation

We define the total density operator of the open system as $\chi(t) = e^{-iHt} \chi(0) e^{iHt}$ which, after the transformation in Eq. (7), can be written in the interaction picture as^{42,44} $\tilde{\chi} = e^{i\bar{H}_0 t} \bar{\chi}(t) e^{-i\bar{H}_0 t}$, with $\bar{\chi}(t) = e^{-i\bar{H}t} \bar{\chi}(0) e^{i\bar{H}t}$. By taking the partial trace over the reservoir degrees of freedom, the reduced density matrix (RDM) of the two DQDs plus the boson bath is obtained as $\tilde{\rho}(t) = Tr_{res} \tilde{\chi}(t)$. Applying the second order Born approximation we obtain the equation of motion for $\tilde{\rho}(t)$ as:

$$\begin{aligned}
\frac{d}{dt}\tilde{\rho}(t) = & -i[\tilde{H}_T(t), \tilde{\rho}(t)] \\
& - \sum_{k,i \in 1,2,j \in L,R} \int_0^t dt' |V_k^j|^2 f^j(\epsilon_k^j) e^{i\epsilon_k^j(t-t')} \{ \tilde{s}_{j,i}(t) \tilde{s}_{j,i}^\dagger(t') \tilde{\rho}(t') - \tilde{s}_{j,i}^\dagger(t') \tilde{\rho}(t') \tilde{s}_{j,i}(t) \} \\
& - \sum_{k,i \in 1,2,j \in L,R} \int_0^t dt' |V_k^j|^2 [1 - f^j(\epsilon_k^j)] e^{-i\epsilon_k^j(t-t')} \{ \tilde{s}_{j,i}^\dagger(t) \tilde{s}_{j,i}(t') \tilde{\rho}(t') - \tilde{s}_{j,i}^\dagger(t') \tilde{\rho}(t') \tilde{s}_{j,i}(t) \} \\
& - \sum_{k,i \in 1,2,j \in L,R} \int_0^t dt' |V_k^j|^2 f^j(\epsilon_k^j) e^{i\epsilon_k^j(t-t')} \{ \tilde{\rho}(t') \tilde{s}_{j,i}(t') \tilde{s}_{j,i}^\dagger(t) - \tilde{s}_{j,i}^\dagger(t') \tilde{\rho}(t') \tilde{s}_{j,i}(t) \} \\
& - \sum_{k,i \in 1,2,j \in L,R} \int_0^t dt' |V_k^j|^2 [1 - f^j(\epsilon_k^j)] e^{-i\epsilon_k^j(t-t')} \{ \tilde{\rho}(t') \tilde{s}_{j,i}^\dagger(t') \tilde{s}_{j,i}(t) - \tilde{s}_{j,i}(t) \tilde{\rho}(t') \tilde{s}_{j,i}^\dagger(t') \}
\end{aligned} \tag{14}$$

where $f^j(\epsilon_k^j) = \text{Tr}_{res}\{R_0 c_{k,j}^\dagger c_{k,j}\}$ are the Fermi distributions of each contact (R_0 is the density matrix of the electron reservoirs, considered in thermal equilibrium).^{42,44,46} Eq. (14) can be simplified by rewriting the sums over k as integrals $\sum_k |V_k^j|^2 f^j(\epsilon_k^j) e^{i\epsilon_k^j(t-t')} = \int_{-\infty}^{\infty} \frac{d\epsilon}{2\pi} \Gamma_j(\epsilon) f^j(\epsilon) e^{i\epsilon(t-t')}$, where $\Gamma_j(\epsilon) \equiv 2\pi \sum_k |V_k^j|^2 \delta(\epsilon - \epsilon_k^j)$ are the tunneling

rates in and out of the DQD. Working in an "infinite bias regime" between the reservoirs (such that $f^L \rightarrow 1$ and $f^R \rightarrow 0$) and assuming a constant density of states in the reservoirs, the coupling with the leads becomes Markovian: $\sum_k |V_k^L|^2 f^L(\epsilon_k^L) e^{i\epsilon_k^L(t-t')} = \Gamma_L \delta(t-t')$ and $\sum_k |V_k^R|^2 [1 - f^R(\epsilon_k^R)] e^{i\epsilon_k^R(t-t')} = \Gamma_R \delta(t-t')$ and, therefore, Eq. (14) reads

$$\begin{aligned}
\frac{d}{dt}\tilde{\rho}(t) = & -i[\tilde{H}_T(t), \tilde{\rho}(t)] \\
& - \frac{\Gamma_L}{2} \sum_{i \in 1,2} \{ \tilde{s}_{L,i}(t') \tilde{s}_{L,i}^\dagger(t') \tilde{\rho}(t') - 2\tilde{s}_{L,i}^\dagger(t') \tilde{\rho}(t') \tilde{s}_{L,i}(t') + \tilde{\rho}(t') \tilde{s}_{L,i}(t') \tilde{s}_{L,i}^\dagger(t') \} \\
& - \frac{\Gamma_R}{2} \sum_{i \in 1,2} \{ \tilde{s}_{R,i}^\dagger(t') \tilde{s}_{R,i}(t') \tilde{\rho}(t') - 2\tilde{s}_{R,i}(t') \tilde{\rho}(t') \tilde{s}_{R,i}^\dagger(t') + \tilde{\rho}(t') \tilde{s}_{R,i}^\dagger(t') \tilde{s}_{R,i}(t') \}
\end{aligned} \tag{15}$$

As we mentioned already, the fact that the coupling with the reservoirs becomes Markovian in this limit implies, in particular, that the renormalization of tunneling due to the bosonic bath becomes ineffective (for example, $\tilde{s}_{L,1}(t') \tilde{s}_{L,1}^\dagger(t') = s_{L,1}(t') s_{L,1}^\dagger(t')$).

Invariance under unitary operations implies that the expected value of any dot operator can be written as $\langle O(t) \rangle = \text{Tr}_{dot}\{\text{Tr}_B\{\tilde{\rho}(t)\} \text{Tr}_B\{\tilde{O}(t)\}\} = \text{Tr}_{dot,B}\{\tilde{\rho}(t) \tilde{O}(t)\}$, where Tr_B is the trace over the bath states. In particular, the expected value of the projec-

tor operators over the system states $Y_{nm} = |n\rangle\langle m|$, can be written as $\langle Y_{nm} \rangle = \text{Tr}_{dot}\{\rho^S Y_{nm}\} = \langle m|\rho^S|n\rangle = \text{Tr}_{dot}\{\tilde{\rho}^S \tilde{Y}_{nm}\}$, where we have defined the RDM of the DQDs array (system) as $\rho^S = \text{Tr}_B \rho$. It is therefore possible to obtain matrix elements of the *reduced density operator* by just calculating the expectation value for the suitable $\tilde{Y}_{nm}(t)$ operators *directly* from the master equation (15). Using the notation $\rho_{mn}^S \equiv \langle m|\rho^S|n\rangle$, we obtain the following set of exact equations:

$$\begin{aligned}
\rho_{nm}^S(t) = & \rho_{nm}^S(0) - i \int_0^t \text{Tr}_{dot,B}\{\tilde{\rho}(t') [\tilde{Y}_{nm}(t), \tilde{H}_T(t')]\} dt' \\
& - \frac{\Gamma_L}{2} \int_0^t \text{Tr}_{dot,B}\{(\tilde{s}_L(t') \tilde{s}_L^\dagger(t') \tilde{\rho}(t') - 2\tilde{s}_L^\dagger(t') \tilde{\rho}(t') \tilde{s}_L(t') + \tilde{\rho}(t') \tilde{s}_L(t') \tilde{s}_L^\dagger(t')) \tilde{Y}_{mn}(t)\} dt'
\end{aligned}$$

$$-\frac{\Gamma_R}{2} \int_0^t Tr_{dot,B} \{ (\tilde{s}_R^\dagger(t') \tilde{s}_R(t') \tilde{\rho}(t') - 2\tilde{s}_R(t') \tilde{\rho}(t') \tilde{s}_R^\dagger(t') + \tilde{\rho}(t') \tilde{s}_R^\dagger(t') \tilde{s}_R(t')) \tilde{Y}_{mn}(t) \} dt'. \quad (16)$$

The full expression of the density matrix is too large to give it here (its total dimension is 6×6) and we show

just two examples for $\rho_{14}^S(t)$ and $\rho_{13}^S(t)$ elements:

$$\begin{aligned} \dot{\rho}_{13}^S(t) &= -i\{t_c e^{-i(\bar{\varepsilon}_1 - \bar{\varepsilon}_3)(t-t')} (\langle \tilde{Y}_{33}(t') X_t^\dagger X_{t'} \rangle - \langle \tilde{Y}_{11}(t') X_{t'} X_t^\dagger \rangle + \langle \tilde{Y}_{32}(t')^\dagger X_t^\dagger X_{t'} \rangle - \langle \tilde{Y}_{41}(t') X_{t'} X_t^\dagger \rangle) \\ \dot{\rho}_{14}^S(t) &= -i\{t_c e^{-i(\bar{\varepsilon}_1 - \bar{\varepsilon}_4)(t-t')} (\langle \tilde{Y}_{43}^\dagger(t') X_{t'} X_t^\dagger X_{t'}^\dagger \rangle + \langle \tilde{Y}_{21}^\dagger(t') X_{t'} X_{t'} X_t^\dagger X_{t'} \rangle \\ &\quad + \langle \tilde{Y}_{42}^\dagger(t') X_{t'} X_t^\dagger X_{t'}^\dagger X_{t'} \rangle - \langle \tilde{Y}_{31}^\dagger(t') X_{t'} X_{t'} X_t^\dagger X_{t'}^\dagger \rangle), \end{aligned} \quad (17)$$

with $\bar{\varepsilon}_n|n\rangle = \left(\sum_i \frac{1}{2} \Delta \varepsilon_i \sigma_z^i - \frac{1}{4} \kappa \sum_{i,j} \sigma_z^i \sigma_z^j\right) |n\rangle$.

Note that Eqs. (17) are not closed. They contain expectation values involving products of dot and boson operators, as for example $\langle \tilde{Y}_{33}(t') X_t^\dagger X_{t'} \rangle = Tr_{dot,B} \{ \tilde{\rho}(t') \tilde{Y}_{33}(t') X_t^\dagger X_{t'} \}$, which need to be decoupled. If one is not interested in the system back action on the bath, the latter can be assumed to remain at thermal equilibrium at all times.⁴² Therefore, the reduced density operator can be approximated as $\tilde{\rho}(t') \approx \rho_B(0) \otimes Tr_B \tilde{\rho}(t')$. By using this approximation, we can decouple higher order correlation functions as $\langle \tilde{Y}_{33}(t') X_t^\dagger X_{t'} \rangle \approx \langle \tilde{Y}_{33}(t') \rangle \langle X_t^\dagger X_{t'} \rangle$, etc. This decoupling corresponds to the so-called Non Interacting Blip approximation in the spin-boson problem⁵⁰.

For an equilibrium boson bath one can write the correlation functions as⁴² $C(t-t') = \langle X_t X_{t'}^\dagger \rangle = e^{-\Phi(t-t')}$ with $\Phi(\tau) = \int_0^\infty \frac{J(\omega)}{\omega^2} \{ (1 - \cos(\omega\tau)) \coth(\beta\omega/2) + i \sin(\omega\tau) \}$. Note that in our problem we also need $C_2(t-t') = \langle X_t X_{t'} X_{t'}^\dagger X_t^\dagger \rangle = e^{-2\Phi(t-t')}$, which appear from coherences involving interdot processes like $|1\rangle \leftrightarrow |4\rangle$ (namely, $|L_1, L_2\rangle \leftrightarrow |R_1, R_2\rangle$, see Eq. (17)). In principle, terms involving half phases $e^{-\frac{1}{2}\Phi(t-t')}$ also appear in tunneling processes to the reservoirs (like, for example, $|R_1, L_2\rangle \xrightarrow{\Gamma_R} |0_1, L_2\rangle$) but, again, they do not contribute in the Markovian limit.

The resulting set of coupled equations can be given in matrix form as

$$\rho_S(t) = \rho_S(0) + \int_0^t \{ \mathbf{M}(t-t') \rho_S(t') + \mathbf{\Gamma} \} dt' \quad (18)$$

where ρ_S is a vector containing the different matrix elements of the reduced density operator, the vector $\mathbf{\Gamma}$ contains the terms related with the coupling of the first DQD to the reservoirs, and $\mathbf{M}(t-t')$ is a non-Markovian time dependent kernel which contains the bath correlation functions.⁴³ Equation (18) can be solved in the Laplace space^{42,43} as

$$\rho_S(z) = [z - z\mathbf{M}(z)]^{-1} (\rho_S(0) + \mathbf{\Gamma}/z). \quad (19)$$

The kernel $\mathbf{M}(z)$ contains the Laplace transform of the bath correlation functions $C_\varepsilon^{(*)} = \int_0^\infty e^{-z\tau} e^{(-)i\varepsilon\tau} C^{(*)}(\tau) d\tau$ evaluated at different energies corresponding to the involved transition.^{42,43,44}

III. ENTANGLEMENT

The full time-dependent density matrix can be obtained by solving algebraically Eq. (19) and performing an inverse Laplace transformation, which is a formidable task. Fortunately, the entanglement generated by the bath is finite at long times, namely in the *stationary state*, as we will show. The stationary solution of Eqs. (18), ρ_∞ , is obtained by extracting the $1/z$ coefficient in a Laurent series of $\rho_S(z)$ for $z \rightarrow 0$.^{42,43,44} For entanglement quantification we use Wootters' concurrence⁴⁸ for a general state of two qubits,

$$C = \max\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\} \quad (20)$$

where the λ s are the eigenvalues in decreasing order of the non hermitian matrix $\rho_S(\sigma_y \otimes \sigma_y) \rho_S^*(\sigma_y \otimes \sigma_y)$. The concurrence ranges from $C = 0$ for non-entangled states to $C = 1$ for the maximum degree of entanglement. That maximum entanglement is showed by the Bell states⁴⁸. In the basis of triplet and singlet states, $|S_0\rangle = \frac{1}{\sqrt{2}}(|L_1 R_2\rangle - |R_1 L_2\rangle)$, $|T_0\rangle = \frac{1}{\sqrt{2}}(|L_1 R_2\rangle + |R_1 L_2\rangle)$, $|T_+\rangle = |L_1 L_2\rangle$ and $|T_-\rangle = |R_1 R_2\rangle$, the Bell states read: $|\Psi^+\rangle = |T_0\rangle$, $|\Psi^-\rangle = |S_0\rangle$, $|\phi^+\rangle = \frac{1}{\sqrt{2}}(|T_-\rangle + |T_+\rangle)$ and $|\phi^-\rangle = \frac{1}{\sqrt{2}}(|T_-\rangle - |T_+\rangle)$.

Importantly, the stationary density matrix in our problem corresponds to a transport situation and, therefore, a proper generalization of concurrence to *non-equilibrium* is needed. Following Ref. 17, we quantify non-equilibrium entanglement via the concurrence C of the stationary state $\hat{P}\rho_\infty$, where \hat{P} is the projection onto doubly occupied states including proper normalization. The projection \hat{P} corresponds to taking the limit

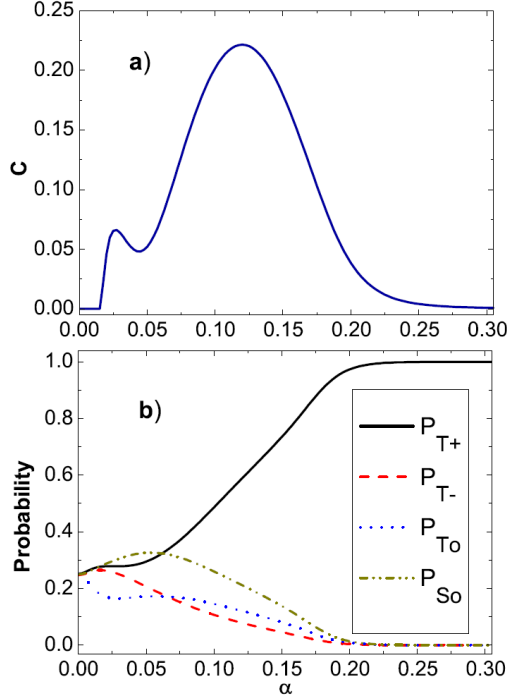


FIG. 2: (Color online) a) Concurrence as a function of the strength of dissipation α . b) Population of triplet and singlet states. Parameters: $\Delta\varepsilon_1 = \Delta\varepsilon_2 = 0$, $t_c = 3.5$, $\Gamma_L = 10$, and $\omega_c = 500$ (in units of $\Gamma_R = 1\mu eV$). These parameters correspond to typical experimental values in AlGaAs-GaAs lateral DQDs^{37,49,62}.

$\Gamma_L \rightarrow \infty$ where both qubits are always occupied with one single electron. For concreteness, we focus on the zero-temperature case.

The concurrence as a function of the coupling α always shows the same qualitative behavior: for very small α there is a switching behavior, indicating that below a minimum interaction strength κ the concurrence vanishes, cf. Fig. 2(a) for identical QDs ($\Delta\varepsilon_1 = \Delta\varepsilon_2 = 0$). As α increases, two effects compete: entanglement generation and localization induced by the bath. At small α , the two delocalized states $|S_0\rangle$ and $|T_0\rangle$ have a finite weight which depends in a nontrivial way on the ratio $\frac{t_c}{\alpha}$. On the other hand, for strong coupling the bath completely freezes the charges on the left dots and the triplet $|T_+\rangle = |L_1, L_2\rangle$ becomes fully occupied, cf. Fig. 2(b). These two competing effects lead to the non-monotonic behavior of the concurrence vs. α , with an optimal value at which the concurrence presents a maximum.

The population of each Bell state is shown in Fig. 3. The system does not originate a preferred Bell state and therefore both maxima in the concurrence contain contributions from all states. The first concurrence peak is formed by a combination of the four Bell states with a symmetric contribution of $|\Psi^+\rangle$ and $|\phi^+\rangle$, whereas on the second peak $|\phi^-\rangle$ probability is slightly dominant. Elec-

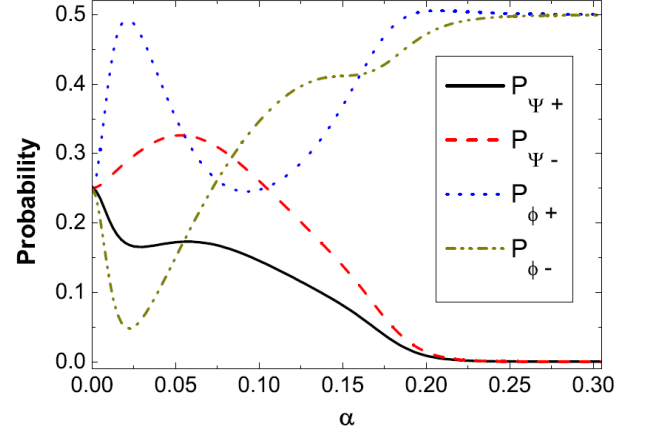


FIG. 3: (Color online) Population of the Bell states as a function of α . Same parameters as Fig. 2.

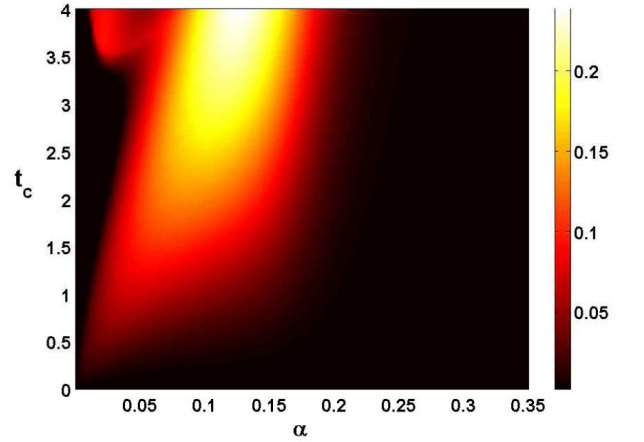


FIG. 4: (Color online) Color map of concurrence vs. interdot tunneling t_c and α . The rest of parameters are the same as in Fig. 2.

trons localization in "parallel" charge states is reflected in the large probability for both $|\phi^+\rangle$ and $|\phi^-\rangle$ states for $\alpha \geq 0.2$.

The concurrence as a function of both t_c and α is shown in Fig. 4. For $2t_c < \Gamma_R$, the dephasing induced by the leads suppresses interdot coherence and the contribution of the delocalized states $|S_0\rangle$ and $|T_0\rangle$ is negligible. Thus, the concurrence is almost zero for all α . For $2t_c > \Gamma_R$, entanglement is finite in a region $\alpha_{min} < \alpha < \alpha_{max}$; both, α_{min} and α_{max} increase with t_c . For $2t_c \gg \Gamma_R$, the system present a maximum in the concurrence at $\alpha \approx 0.15$ with values $C \approx 0.3$.

The effect of Γ_R on concurrence is shown in Fig. 5. Here, we also find the switching behavior described above: starting from $\alpha = 0$, the state of the system is strongly mixed for small Γ_R . Therefore, $C = 0$ below a minimal value α_{min} . This threshold value decreases as

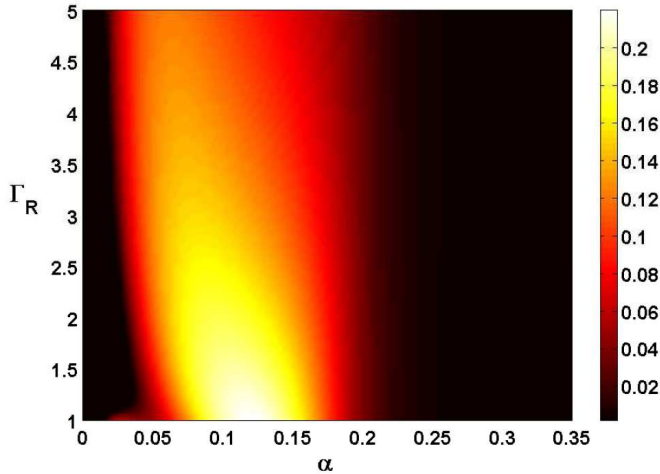


FIG. 5: (Color online) Color map of concurrence vs. tunneling rate to the right lead Γ_R and α . The rest of parameters are the same as in Fig. 2.

Γ_R increases. At fixed α , the entanglement decreases as one increases Γ_R . For very large Γ_R , the pure localized triplet $|T_+\rangle = |L_1, L_2\rangle$ is reached and thus the entanglement is zero. This effect, which is a transport version of the Quantum Zeno effect, is similar to the one occurring in capacitively coupled charge qubits open to reservoirs.¹⁷

A finite detuning $\Delta\varepsilon_i > 0$ ($\Delta\varepsilon_i < 0$) localizes the charge on the lower (upper) QD of each pair and, therefore, the entanglement should depend on whether $\Delta\varepsilon_1 = \Delta\varepsilon_2 > 0$ or $\Delta\varepsilon_1 = -\Delta\varepsilon_2 > 0$. The concurrence of the latter case is very similar to the one for $\Delta\varepsilon_1 = \Delta\varepsilon_2 = 0$, and therefore the population of singlet and triplet states show also the same kind of behavior, Fig. 7(a). On the contrary, the concurrence for $\Delta\varepsilon_1 = \Delta\varepsilon_2 > 0$ is different with a narrow resonance at small α , cf. Fig. 6. This resonance corresponds to a maximum in the population of the triplet T_0 , cf. Fig. 7(b), followed by a fast decay of both T_0 and S_0 and an enhanced population of T_+ (and, hence, zero concurrence). The overall qualitative behavior is in agreement with Ref. 20 where the current through two DQDs coupled to the same phonon bath is analyzed in the Markovian limit. The indirect interaction due to bath leads to an enhancement of the inelastic current at $\Delta\varepsilon_1 = \Delta\varepsilon_2 > 0$, and a maximum population of the triplet T_0 , which is a transport version of the Dicke effect. In close analogy with the Dicke effect in quantum optics, this superradiance should turn into subradiance as the probability of finding the system in the singlet S_0 , rather than in the triplet T_0 , increases.²⁰ We do not find, however, the subradiance counterpart in our analysis of concurrence.

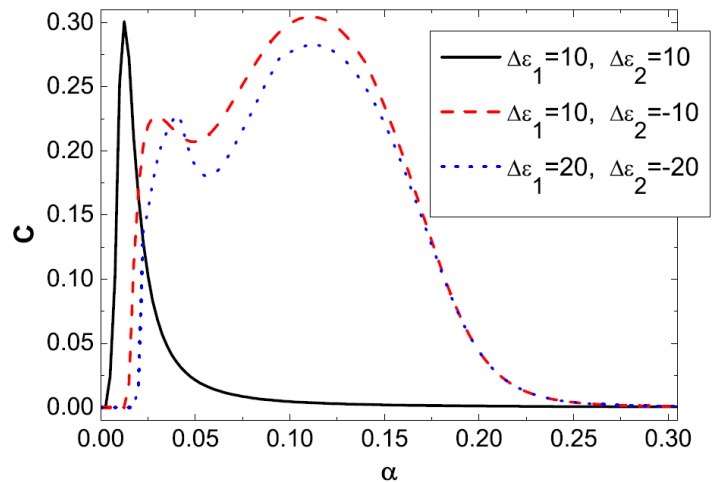


FIG. 6: (Color online) Concurrence as a function of α for different level detunings. The rest of parameters are the same as in Fig. 2.

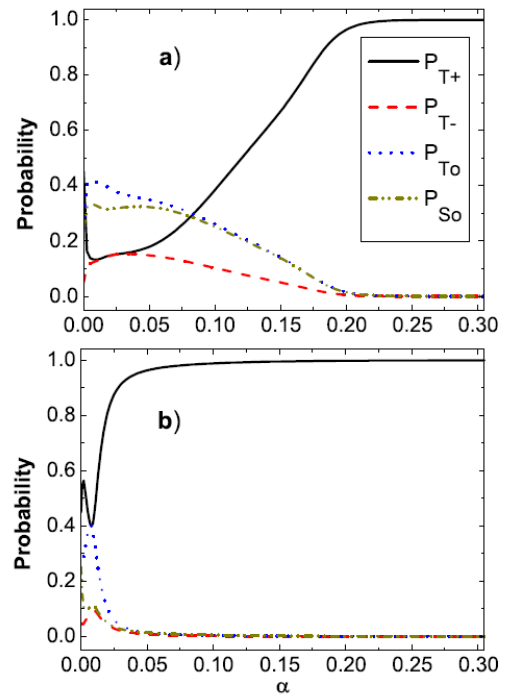


FIG. 7: (Color online) a) Population of triplet and singlet states for $\Delta\varepsilon_1 = \Delta\varepsilon_2 = 10$. b) The same for $\Delta\varepsilon_1 = \Delta\varepsilon_2 = -10$.

IV. CONCLUSION

We have shown that the strong coupling of two independent qubits with a common bosonic bath at zero temperature originates entanglement between the qubits in the *stationary limit*. We also identify that two ef-

fects compete as a function of the coupling strength with the bath: entanglement generation and charge localization induced by the bath. These effects lead to a non-monotonic behavior of the concurrence as a function of the coupling strength with the phonons.

In addition, the concurrence strongly depends on tunneling and energy difference on each DQD as well as on the coupling with external leads, parameters which can be controlled experimentally.

Due to the small concurrence values obtained here ($C < 0.5$), and the fact that no preferred Bell state is formed, the use of this setup may not be an optimal choice for entanglement studies in the solid state realm. Note, however, that this system is the minimal implementation of a fully tunable two qubit system coupled

to a common bath. From this point of view, this realization is an attractive benchmark in which to study the interplay between quantum coherence, entanglement and decoherence.

Acknowledgments

We acknowledge F. Rojas and E. Cota for fruitful discussion. DC was funded by DGAPA-UNAM (project IN114403) and CONACyT ("beca mixta" funds and project 43673-F). RA acknowledges financial support from grants MAT2006-03741 (MEC-Spain), 20060I003 (CSIC) and 200650M047 (CAM).

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